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A biological junction with quantum-like characteristics

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A model of chemical synapse as an electric junction is proposed. Estimations and analysis of the model show that the junction has unique physical characteristics reminding the Josephson junction. The basic assumption is made that the electric coupling across the synaptic gap is indirectly provided by means of approximately quantized portions of a chemical mediator, each the portion is content of a synaptic bubble. We suppose that effective quantum of charge is q , $|q| \gg |e|$. The synapse characteristics are dominated by electrostatic energy, $Q^2/2C$, $Q = qN$, $N = 0, 1, 2, \dots$; where C is electric capacity of membrane. Estimations show that the integer-valued character of N must be explicitly taken into account. The consistent theory of the junction is constructed on the basis of operator realization of number-phase canonical pair in the Hardy space. The charge passing from one side of the junction to other is described by the Toeplitz operators. The synapse state space is constructed explicitly. The unique physics of the model is investigated in detail. We do not exclude the possibility that the model is prototype of a molecular electronics device.

**A model of mesoscopic junction:
The benefits of number-phase operators
(application in biophysics)**

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In this talk we present a primary approach to the physical modeling of a chemical synapse as the quantum-like junction in a nearly closed loop of a self-synapsing neuron. We consider such kind of biological systems as a candidate for detecting and processing of microwave radiation. Explicit realization of the state space and the junction Hamiltonian operator are constructed consistently.

The influence of electromagnetic waves, especially of microwave radiation, on biological systems has attracted much interest for a long time. In particular, active areas of research have been the study of the results of radio- and microwave radiation effect in biological populations, or the possible influence of usual household electrical appliances of the new generation on humans and others living systems, or an accident prevention in industry. These investigations are of interest mainly from a phenomenological viewpoint since the works on cellular or molecular level are rare up to now. They are also of interest from the point of view of understanding how the electromagnetic waves of different frequency and power affect biological systems. By the way, there were the rumors that in the former USSR the building of the USA Embassy in Moscow has been exposed to low-level microwave radiation during of many years. On the other hand, it seems that the interest of physicists in this subject is concentrated also in the field of macroscopic electrodynamics phenomenology, being quite less rich than electrodynamics of complex nonequilibrium cellular- and molecular-level biological systems.

At the present time, at least in Donetsk, Ukraine, there is the official medical institution for therapy and research, Hospital "Sitko" – MRT (microwave therapy), where treat patients with extremely low-level microwave radiation by the special procedures. Thus, some aspects of the microwave therapy are clinically tested.

The present work was originally motivated by the desire to understand a possible mechanism of the effect of a low-level microwave radiation on biological objects such as humans (or rats, for example), – and on a cellular level, beyond the phenomenology. It is in connection with the microwave therapy. On the other hand, the desire was inspired by the recent works in the field of neurophysiology. Namely, our model accepts a hypothesis which is based on the existence of in a sense mesoscopic self-synapsing neurons, neurological loops with chemical synapses. The experimental evidence that such the loops are really existing has been recently reported [1] (concerning young rats).

In the present talk, we set up the physical model and the basic theoretical framework we shall need for the study of detection and processing of extremely low-level microwave radiation by the biological systems on the cellular level. The results we present here will be used in the following works.

Before starting to work out the physical model and its relations with the biological system, it may be worth getting some idea of what it is about, in particular what is a synapse and what is a junction. Of course, in a schematic, without any details, way.

Above all, let us to give some preliminary idea about the neurological system under consideration. A typical neuron has about $10^3 \div 10^4$ synapses. Self-synapsing neuron is a nearly closed, circular loop of electrically excitable (neuron membrane) conducting material, the biological loop of axon and its dendrite, of in a sense mesoscopic size (to be defined below). The thickness of a neuron membrane is about 50\AA and the electric capacity of membrane is about $1\mu F/cm^2$. The loop contains a gap. It is the gap of a synapse (usually $\sim 10^2\text{\AA}$). We suppose that it is a chemical synapse. It means that the electrical coupling across the synaptic gap is indirectly provided by means of quantized portions of chemical mediator. Each the portion, the content of a synaptic bubble, contains about 10^3 molecules of a chemical mediator. We suppose that an effective "quantum" of charge in synapse is about 10^3e . For further relevant and more detailed information concerning neurons, synapses and all that we refer the reader to [2].

Then, the notion 'mesoscopic junction' is generic for a wide class of the physical systems. The tunnel junction is a prototype the junction class, of course the last has the matter far beyond this, that illustrates the relevant physical phenomena and the corresponding theoretical prolegomena. Physical systems like mesoscopic junctions are widely established in current physical literature.

Here, beginning with the simple physical model of a complex biological junction, we construct the consistent quantum-like theory of mesoscopic junctions including the explicit realizations of the model state space and the Hamiltonian operator as an operator in this space, and in what follows we describe in this frame the corresponding dominant physical effects together with application to the biological system, such as a self-synapsing neuron, which is treated as a candidate on the role of detector and processor of microwave radiation in biological systems.

The physical model consists in the following. A self-synapsing neuron system is idealized as a circular loop of electrically conducting material. The synaptic gap in this loop is modeled as a junction of relatively small capacity C . Really it is a system of two membrane capacities connected by physiological solvent; however, it is easy to find the arguments that we can replace this system by single effective capacity (e.g., the resistance across a synapse is dominated by the membranes). But, on the other hand, the charge carriers in this junction are "quantized" due to chemical nature of the synapse. It means that an effective elementary charge Q may be considerably greater than the charge of electron and every additional charge Q will change electrostatic energy on the junction substantially. Under the such conditions the role of charge energy on the junction increase and we must to take the quantum-like nature of the effective charge into account. All that is the first part of the system mesoscopicity condition mentioned above. The second one is the geometric size (radius) of the loop. This second aspect is connected with the fact that magnetic fields

penetrate biological tissue much more effectively than electric fields and thus the geometric size of the loop is directly connected with magnetic flux through the loop, and will dominate in detection and processing of microwave radiation, for example, by means of a depolarization of the membrane and an induced exit of a mediator into the synaptic gap. But it is the topic of another paper.

In order that the model be more mathematically formulated, it is sufficient to define the character of relevant macrovariables. We set the number N of Q carriers as the characteristic macrovariable. Further, we assume that in respect of the characteristic macrovariable an homogeneous state on the junction is realized. And also we take into account the discreteness ("quantization" by Q) of a charge magnitude on the junction explicitly. This implies that the relevant is setting as fundamental the canonical pair of the action-angle (number-phase) operators realized on a proper state space; we realize the state space as the Hardy space \mathbf{H}^2 (e.g., [3]).

In this point the principal from the theoretical point of view and crucial for the theory question is arising: How much a wealth of material can be extracted from the model to be restricted to the fact of discreteness of a charge carriers and under conditions of a system mesoscopicity (e.g., concerning electric capacity, inductance or geometric size)? The answer to this question give us the key to a lot of the problems.

Then, there are usual arguments that after a coarse-graining procedure a quantum-like energy operator, the Hamiltonian operator H , is a function of the variable N only. By the way, it is in perfect harmony with Ginzburg–Landau phenomenology. Indeed, if the canonical pair of operators (N, Φ) , $[\Phi, N] = i$, is defined in the Hardy space \mathbf{H}^2 (it is reasonable way) then using isometry $\mathbf{H}^2 \rightarrow \mathcal{L}^2$ and the Wigner phase-space representation together with the corresponding formula:

$$\begin{aligned} \text{Tr} \exp(-\beta H) &= \int d^2\psi \exp(-\beta F(\psi)), \\ \exp(-\beta F(\psi)) &\equiv 2 [\exp(-\beta H)]_W(\psi), \quad \psi \in \mathbf{C}^1, \end{aligned}$$

where $[\cdots]_W$ denotes the Wigner–Weyl symbol of the corresponding operator (in \mathcal{L}^2), – and with identifying $F(\psi)$ as the Ginzburg–Landau free energy, we obtain $F(\psi) = F_0 + A|\psi|^2 + \frac{1}{2}B|\psi|^4 + \cdots$, where F_0, A, B, \dots are explicitly given if there is given the operator H . Inversely, by a given free energy $F(\psi)$ we obtain $H = H(N) = H_0 - \beta^{-1} \sum_{(n)} ((-1)^n / n!) K_n N^n$, where the coefficients are explicitly given if there is given the function $F(\psi)$; $\{K_n\}$ have the structure of cumulants. Note, that evaluation procedure of this point is of interest in its own right.

Let us now return back to model of the biological junction together with the number of Q carriers as the distinctive variable, and start from the problem of two sides, 1 and 2, of a synapse coupled by this junction. Firstly, if we neglect the coupling between sides 1 and 2, the Hamiltonian operator breaks into two parts $H_1 + H_2$. Further, if no external voltage is applied on the junction, the chemical potential on the sides 1 and 2 are equal. It means that between the states (N_1, N_2) and $(N_1 - \nu, N_2 + \nu)$, $\nu \in \mathbf{Z}$, no difference.

Let us now allow a coupling between 1 and 2. It can be split into two parts: (1) electrostatic, with a capacity C ; (2) a charge passing from one side to other with the extracting, for example, of Q in 1 and bringing it in 2.

Let us now realize the state spaces of 1 and 2 as the Hardy spaces \mathbf{H}^2 and define in these spaces the pairs of the number-phase operators (N_1, Φ_1) and (N_2, Φ_2) , $[\Phi_k, N_l] = i\delta_{kl}$, $k, l = 1, 2$; together with the Toeplitz partially isometric one-sided translation operators T_+^1, T_-^2 . And let $\{e_n^{(1)}\}$ and $\{e_n^{(2)}\}$ are the standard basis in \mathbf{H}_1^2 and \mathbf{H}_2^2 correspondingly. In this case we can construct the state space as $\mathcal{H} = \mathbf{H}_1^2 \otimes \mathbf{H}_2^2$, and define the following operators

$$N_0 = N_1 \otimes 1 + 1 \otimes N_2, \quad N = N_1 \otimes 1 - 1 \otimes N_2, \\ \Phi = \frac{1}{2} (\Phi_1 \otimes 1 - 1 \otimes \Phi_2),$$

with the commutation relations

$$[\Phi, N_0] = 0, \quad [\Phi, N] = i,$$

on a dense domain in \mathcal{H} . We suppose also that N_0 is fixed. The pair (N, Φ) is principal set of operators. It is easy to see that it is convenient to take \mathbf{H}_-^2 instead of \mathbf{H}_2^2 , where \mathbf{H}_-^2 is the subspace of $\mathbf{L}^2 = \mathcal{L}^2(C_1, d\varphi/2\pi)$ spanned on $\{e_{-n}\}_0^\infty$. It implies some evident overdetermination.

At a given N_0 , N can takes $(2N_0 + 1)$ values. Under this condition we can explicitly realize \mathcal{H} as the subspace of the Laurent space \mathbf{L}^2 . In this way we will be prepared to take down a junction Hamiltonian explicitly:

$$H = H_0 + \frac{1}{2C} N^2 + T,$$

where C is electric capacity and T is operator of extracting a charge in 1 and bringing it in 2: $T = t_1 (T_+ + T_-) + t_2 (T_+^2 + T_-^2) + \dots$, – most probably

$$T = t (T_+ + T_-),$$

and we can choose t real.

As first test of the model presented we can make passage to “classical” limit of the corresponding equations of motion. Note, that using the Wigner phase-space representation as well as the coherent states representation we obtain the equations are analogous to known Josephson equations.

In conclusion, we hope that the model has enough wealth of detail relevant to biological insight as well as interesting physics. But that is all for this primary presentation.

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